THICK OR THIN? MODELLING FILM FLOWS FROM MICRONS TO KILOMETERS

Stéphane Popinet[1], Arnaud Antkowiak[1], Pierre-Yves Lagrée[1], Luc Deike[2], Jiarong Wu[2,3] et al.
[1]*CNRS & Sorbonne Université, Institut Jean le Rond ∂'Alembert, Paris, France*
[2]*Princeton University, Mechanical and Aerospace Engineering, Princeton, USA*
[3]*New York University, Courant Institute of Mathematical Sciences, New York, USA*

Abstract

We present a generalised multilayer model, including capillarity, Marangoni stresses and surfactant transport, able to accurately describe free-surface flows at scales ranging from micrometers to thousand of kilometers. Applications will include the instabilities in marginal regeneration in soap bubbles, the statistics of wave breaking in the ocean and the prediction of natural disasters such as tsunamis and avalanches.

Introduction

Many natural and artificial systems have a restricted extent in one of their spatial dimensions. For fluid flows a typical example is the atmosphere or the ocean for which vertical motion is much more constrained than horizontal motion. At the other hand of the scale spectrum, and for an important man-made system, one could think of the flow of oil lubricating the thin gap between a rotating shaft and its supporting metal sleeve.

This basic observation has led many scientists to look for simplified representations of such "thin film" flows. A well-known example of such a model is the "shallow-water" set of equations proposed by Saint-Venant in 1871, for applications to flowing rivers and tidal flows. At the other hand of the scale spectrum, and in the engineering context mentioned above, the "lubrication approximation" is applicable to viscosity-dominated flows.

While these approximations are valid and useful for a range of applications, many important questions require models able to bridge the gap between "thin films" and "thick films". These include for example: the surfactant-driven dynamics of soap films and foams (Figure 1), viscous and inertial gravito-capillary waves (Figure 2), the large-scale statistics of breaking oceanic waves (Figure 3) or the risks due to dispersive waves generated at a geophysical scale (Figure 4).

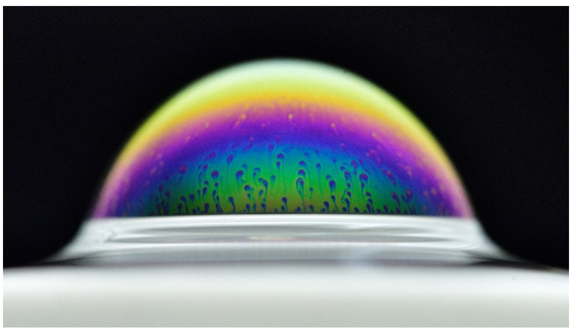

Figure 1: **10 μm**: Marginal regeneration in a soap bubble [2].

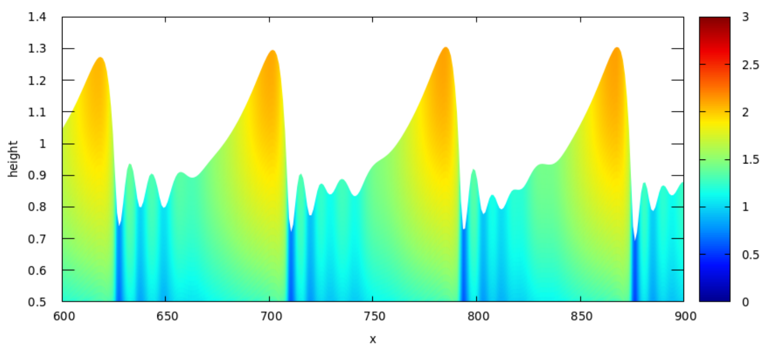

Figure 2: **1 cm**: Kapitza waves [3].

Models

We recently proposed a model able to describe both "thin films" and "thick films" [1] in the context of large-scale geophysical flows. In this presentation, I will give an overview of the principal properties of this multilayer, semi-discrete approximation of the incompressible Navier–Stokes equations with a free-surface and its theoretical and practical connections with previous classical film models.

I will also describe a recent extension of the model to surface-tension and surfactant-driven flows, including the robust numerical treatment of complex surface terms [3]. The model will be shown to recover a range of analytical asymptotic solutions for non-trivial configurations, at very-small scales, such as the unstable viscous drainage in "Plateau borders" ultimately leading to surfactant "convection patterns" in soap films (Figure 1).

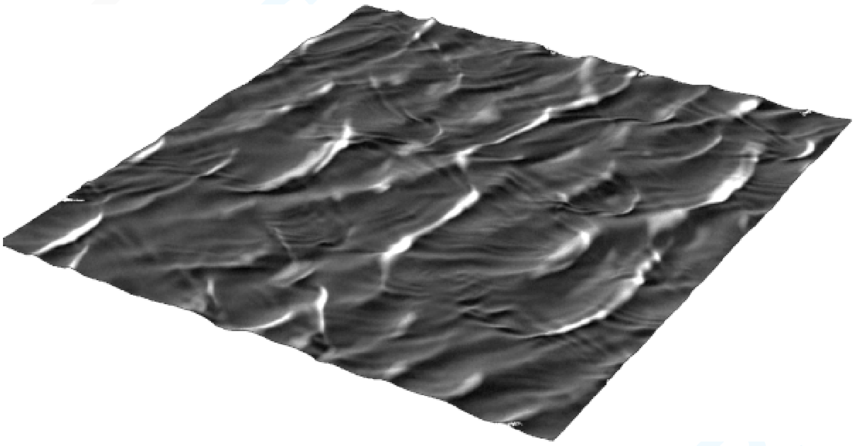

Figure 3: **1 km**: Oceanic breaking waves [4].

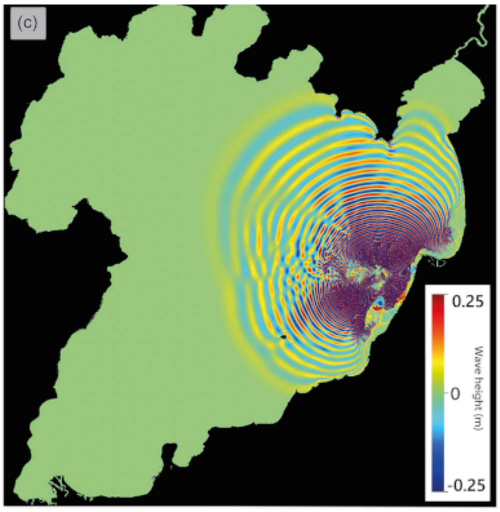

Figure 4: **100 km**: Dispersive waves generated by a volcanic eruption under lake Taupo, New Zealand [5].

At larger scales, the ability of the model to accurately describe vertical accelerations, as well as the detailed three-dimensional structure of turbulent boundary layers, will be demonstrated through the study of the statistics of large-scale wave breaking in the ocean (Figure 3). At still larger scales, the dispersive properties and coupling with non-trivial rheologies (for e.g. granular materials) will illustrate the model's capabilities for the prediction of the risks associated with catastrophic natural events such as tsunamis, avalanches or floods (Figure 4).

References

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